THERMAL CHARACTERISTICS OF A VENTILATED GRANULAR BED

É. L. Bogomaz, A. K. Bondareva,
and O. M. Todes

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The rate of heat removal by a gas current in a ventilated granular bed has been estimated and measured.

1. Effective Characteristics in Heat-Transfer Processes

During stationary or nonstationary heating of a granular bed without through flow, the temperature distribution in space and time is determined by the basic thermal characteristic of the bed as a whole, namely, the effective thermal conductivity λ_e or thermal diffusivity $a = \lambda_e/c\rho_p$; as the main resistance to heat flow is produced by the gas layers between the grains, λ_e is dependent in the main on the thermal conductivity λ_g of the gas together with the porosity ε, being largely independent of the thermal conductivity λ_s of the solid, which is usually larger by 2-3 orders of magnitude than λ_g. As a result, $\lambda_g < \lambda_e \ll \lambda_s$, and for mean porosities $\varepsilon \approx 0.4$ the effective thermal conductivity of an immobile unventilated bed is higher than λ_g by about an order of magnitude.

When λ_e is measured by the instantaneous point-source method, it is assumed that the temperature distribution in space and time T(x, y, z, τ) obeys the equation for nonstationary conduction:

$$c\rho_p \frac{\partial T}{\partial \tau} = \lambda_e \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right]. \tag{1}$$

If the case is one of spherical symmetry, the solution takes the form

$$T(r, \tau) = \frac{Q}{c\rho_p} \cdot \frac{1}{(\sqrt{4\pi a\tau})^3} \cdot \exp\left[-\frac{r^2}{4a\tau} \right], \tag{2}$$

where Q is the total amount of heat released by the instantaneous source. The temperature is measured at a given distance r from the source and gives a curve as in (2) with a maximum; the time when this is reached,

$$\tau_m = \frac{r^2}{6a}, \tag{3}$$

enables one to determine the thermal diffusivity. The maximum temperature rise at this instant is

$$T_m = \sqrt{\frac{6}{\pi e^3}} \cdot \frac{Q}{\frac{4}{3}\pi r^3 c\rho_p} = 0.308\overline{T} \tag{4}$$

and this decreases as the distance from the source increases in inverse proportion to r^3, i.e., to the heated volume.

In this discussion, the local temperatures of the particle and gas in the gaps between the grains are considered as identical; this may be taken as justified if the heat transfer between the grains is fairly rapid. Our measurements of heat waves in ventilated and unventilated granular beds [1] have shown that the increasing spread in the heat wave during

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propagation through the bed is largely due to the effective thermal conductivity. The heat transfer between the gas flow and the grains is fairly rapid, and it has only a secondary effect on the width of the wave. It is therefore quite reasonable to assume on the whole that the local gas and solid temperatures are equal, $T_g = T_s = T$.

All the same, the temperature and heat-flux distributions in a ventilated granular bed are more complex than in an unventilated one. Firstly, the heat is transported by the gas flow itself, and we should introduce a convective term $-c_g\rho_g u(\partial T/\partial z)$ into (1), where u is the speed of the gas flow along the z axis. Secondly, force convection (turbulence) will occur as u increases, and this will accentuate the heat transfer by the gas from particle to particle; i.e., the effective thermal conductivity of the gas in the gaps is $\lambda_g^* > \lambda_g$. As the effective thermal conductivity and thermal diffusivity are proportional to λ_g^*, then a also increases with u.

Then the heat-balance equation for the bed takes the form

$$\frac{\partial T}{\partial \tau} = a\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] - v\,\frac{\partial T}{\partial z}, \tag{5}$$

where

$$v = u\,\frac{c_g\rho_g}{c_s\rho_p} \ll u \tag{6}$$

is the effective rate of heat transport by the gas flow, which is another characteristic in addition to a that effectively represents the heat transport in a ventilated granular bed.

For an instantaneous point source we get that the temperature distribution will be dependent on the two parameters a and v:

$$T(x, y, z, \tau) = \frac{Q}{c\rho_p}\cdot\frac{1}{(\sqrt{4\pi a\tau})^3}\cdot\exp\left[-\frac{x^2+y^2+(z-v\tau)^2}{4a\tau}\right] \tag{7}$$

and to determine these parameters from the observed curve is more difficult than for the case v = 0.

For unvarying a and v at a given point (x; y; z = const) the relationship of (7) is a curve with a peak; the time τ_m to reach this is defined by $\partial T/\partial \tau_m = 0$, which leads to the quadratic equation

$$\frac{r^2}{4a\tau_m} - \frac{3}{2} - \frac{v^2\tau_m}{4a} = 0. \tag{8}$$

Then τ_m for a given $r^2 = x^2 + y^2 + z^2$, i.e., at an identical distance from the source in any direction, is not dependent explicitly on z (on the orientation of the receiver relative to the source). For v = 0 we get automatically from (8) that $\tau_m = r^2/6a$, as from (4) for the case of an unventilated bed. The fall in τ_m as v increases is at first slow, but afterwards is inversely proportional to v, as Fig. 1 shows in the dimensionless coordinates $6a\tau_m/r^2$ and $rv/6a$:

$$\text{for } \frac{rv}{a} \ll 6\ \ \tau_m = \frac{r^2}{6a}\left[1 - \frac{v^2r^2}{36a^2}\right], \text{ and for } \frac{rv}{a} \gg 6\ \ \tau_m \approx \frac{r}{v}.$$

We rewrite (7) as

$$T_m = \frac{Q}{c\rho_p}\cdot\frac{1}{(\sqrt{4\pi a\tau})^3}\cdot\exp\left[-\frac{r^2}{4a\tau_m} + \frac{zv}{2a} - \frac{v^2\tau_m}{4a}\right], \tag{9}$$

to see that T_m for r = constant is explicitly dependent on z and is no longer symmetrical with respect to the detector orientation. The largest value for this maximum rise will occur with the detector placed at z = +r, and the least with z = −r. In the plane of the source, z = 0, one should find an intermediate value. From (9) we get a relation between these quantities:

$$T_{m\,\text{with flow}} : T_{m\,\text{in plane}} : T_{m\,\text{against flow}} = e^{\frac{rv}{2a}} : 1 : e^{-\frac{rv}{2a}}. \tag{10}$$

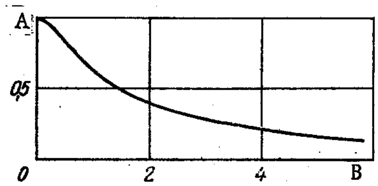

Fig. 1. Time to reach maximal temperature at a distance r behind an instantaneous source as a function of flow speed: $A = 6a\tau_m/r^2$; $B = rv/6a$.

The Peclet parameter

$$\mathrm{Pe} = \frac{rv}{a} \tag{11}$$

appears in (10) in the exponent, and thus even comparatively small values $\mathrm{Pe} \approx 1$ result in a considerable asymmetry in the peak temperatures, whereas the gas speed has little effect on τ_m.

The value of T_m at a given point varies with the speed in accordance with (9) and (8); if we assume that a = constant although v varies, we can calculate the derivative

$$\frac{dT_m}{dv} = \frac{\partial T_m}{\partial \tau_m}\cdot\frac{d\tau_m}{dv} + \frac{\partial T_m}{\partial v} = 0\cdot\frac{d\tau_m}{dv} + \frac{\partial T_m}{\partial v} = \frac{\partial T_m}{\partial v} = T_m\left\{\frac{z}{2a} - \frac{2v\tau_m}{4a}\right\}.$$

This shows that at points in the source plane and upstream in a flow for $z \ll 0$, r always have $dT_m/dv < 0$; i.e., the maximum temperature here should always fall as the flow speed increases. On the other hand, a point directly downstream from the source at $z = +r$ always has $dT_m/dv > 0$, since Fig. 1 shows that $\tau_m \le r/v$; for very large values of $\mathrm{Pe} = rv/a \gg 1$ we get at this point $z = +r$ a comparatively slow and nonexponential rise in T_m with v in accordance with

$$T_m \approx \frac{Q}{c\rho_p}\cdot\frac{1}{\left(\sqrt{4\pi a\frac{\tau}{v}}\right)^3} \sim v^{\frac{3}{2}}.$$

However, it is doubtful whether such large Pe are attainable, since the force convection in the gaps between the grains will increase with the flow speed, and a will consequently increase. If a increases more rapidly than v, the maximum temperature rise at $z = +r$ may begin to fall.

In our previous studies on heat transfer from a gas flow passed through a packed tube [2] we examined a layer of sand, crushed rock, and steel balls of diameter d = 2-6 mm. The Reynolds number for the flow between the grains $\mathrm{Re} = ud/\nu$ varied from 10 to 600; the effective transverse thermal conductivity was deduced indirectly from the relation between the overall heat-transfer coefficient to the walls and the Reynolds number. We found comparatively slow increase in the thermal conductivity of the layer in the working range, namely, as $\lambda_e \sim u^{1/3}$; for $\mathrm{Re} < 1$ we may assume that λ_e should be even less dependent on u. To establish to what extent it is sufficient to use only the two effective characteristics a and v to describe heat transfer in a ventilated or fluidized bed, we made measurements on fine-grained systems at small values of the Reynolds number.

2. Methods

The experiments were done with beds of quartz sand of mean diameter d = 0.3 mm; for subsequent comparison with a fluidized bed, the sand was filled into a vertical column of diameter 80 mm and blown with air from the bottom upwards. The height of the layer was 80 mm, and the packed density was $\rho_p = 1600\ \mathrm{kg/m^3}$, while the specific heat of the sand, and also that of the air, was c = 1 J/kg·deg. The Archimedes number for these particles in air was $\mathrm{Ar} = (gd^3/\nu^2)\cdot(\rho_s/\rho_g) = 2600$. The critical flow speed u_c at which the bed became fluidized can be estimated [3] from $\mathrm{Re}_c = u_c d/\nu = \mathrm{Ar}/1400 \approx 2$, which gives u_c = 10 cm/sec; therefore, we measured the thermal characteristics of the ventilated but immobile bed at u of 5-7 cm/sec, i.e., at $\mathrm{Re} \approx 1$.

The point source of heat must be small by comparison with the scale of the layer, but have dimensions several times the particle diameter, which allows one to consider it as a continuous medium in relation to the granular bed with air gaps between the grains. The heater was a wire loop (Nichrome) of thickness 0.5 mm and turn diameter 2.5 mm, the height of the turn being about 1.5 mm. This loop was mounted in a holder, which also held the thicker supply leads, and was inserted through a hole in the wall of the column to bring the turn to the axis (the origin) and a heavy current with a density of about 100 $\mathrm{A/mm^2}$ was passed for about 0.5 sec.

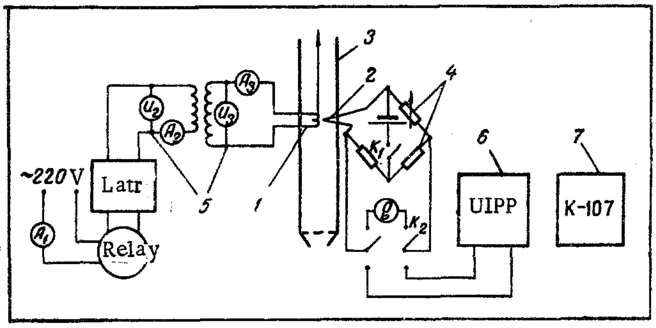

Fig. 2. Block diagram: 1) heater; 2) microthermistor; 3) column; 4) bridge; 5) heat-pulse source; 6) amplifier; 7) oscilloscope.

If we assume the mean radius of the volume in which the heat is released is about 1.5 mm, while the maximum temperature rise is 1000° above room temperature, we get from (4) that at 10 mm from the center of the heater the maximum rise will be lower by a factor $(10/1.5)^3$ and will not exceed 2-3°. We measured the very small rises with an MT-54 microthermistor with a tip of thickness 0.5 mm, which was mounted in a special ebonite holder. This holder was inserted through a hole in the wall opposite the heater insertion. The tip could be placed at r = 10 mm from the heater in three positions: above the heater (z = +r), under the heater (z = −r), and in the same plane (z = 0). With no flow we also used a more remote position at r = 20 mm, at which the expected maximum rise should not exceed 0.5°. The heater and thermistor were set up in the empty column, which was then filled with sand.

Figure 2 shows the block diagram. The line voltage is passed through a timer to an autotransformer, whose secondary is connected to a step-down transformer to produce a current up to 23.5 A (instantaneous reading of ammeter) for τ = 0.5 sec. The resistance of the loop was 0.26 Ω, so the amount of heat supplied to the source did not exceed 70 J, and then (4) indicates that T = 3.5° at r = 10 mm. The variation in the current during the pulse and the heat loss through the leads reduced this in fact to about 1.78°.

The thermistor was connected in an unbalanced bridge, and the voltage arising in the diagonal was passed to a UIPP-2 amplifier and then to a K-107 loop oscillograph. The range in the temperature was small, so the beam deflection was virtually linear as a function of temperature. A certain deficiency of the design was the comparatively thick holder (diameter about 7 mm), which somewhat perturbed the conditions assumed in Sec. 1 (spherical symmetry around the heater). The second assumption about the layer, namely, unbounded dimensions, was reasonably justified with the diameter and height used.

3. Results and Discussion

Figure 3 shows results for various positions of the thermistor and various modes of operation; lines 4 and 5 represent the temperatures with u = 0 at r = 10 mm and r = 20 mm. At this r, the observed points for all three positions (1, 2, and 3) fit to the same curve, and the behavior is well described by a single effective characteristic, namely, a, which is given by (3) as

$$a = \frac{r^2}{6\tau_m} = \frac{(1\ \text{cm})^2}{6 \cdot 44\ \text{sec}} = 3.8 \cdot 10^{-3}\ \frac{\text{cm}^2}{\text{sec}} = 3.8 \cdot 10^{-7}\ \frac{\text{m}^2}{\text{sec}}.$$

Correspondingly, the effective thermal conductivity has the reasonable value $\lambda = a c \rho_p$ = 0.61 W/m·deg; when r was doubled, τ_m increased by about a factor of four, while T_m fell by about a factor of eight. This very small rise did not allow us to obtain reasonably accurate values from the second heavy line 5 in Fig. 3, since the thickness of the line on the recording was comparable with deviation from the baseline.

The thin lines 1-3 in Fig. 3 show the temperatures for u = 5.8 cm/sec recorded with r = 10 mm at all three points (1 above the loop, 2 below the loop, and 3 at the level of the loop). These curves differ considerably on account of the heat transfer along the z axis. We expect from (6) that the second effective heat characteristic, namely, the heat-transport rate along the flow, should be

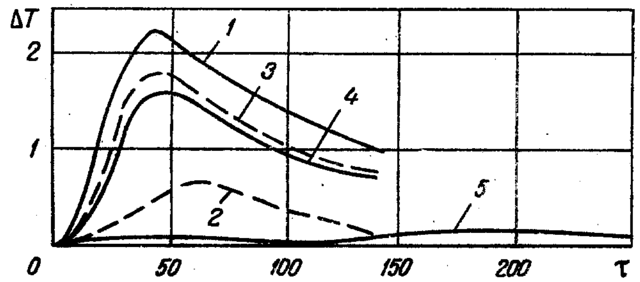

Fig. 3. Heating curves (ΔT, °C; τ, sec) for u = 5.8 cm/sec: 1) along flow; 2) against flow; 3) at heater level; no flow, r (mm) of: 4) 10; 5) 20.

$$v = \frac{c_g \rho_g}{c_g \rho_p} u = 5.8\,\text{cm/sec} \cdot \frac{1 \cdot 1.3}{1 \cdot 1600} = 4.7 \cdot 10^{-3}\,\text{cm/sec}$$

and Pe = rv/a = $(4.7 \cdot 10^{-3} \cdot 1)/(3.8 \cdot 10^{-3})$ = 1.24 is of the order of one; as we have seen in Sec. 1, the position of τ_m on all three curves at r = constant should be the same and dependent solely on a; on the other hand, the T_{mi} should be different: T_{m1} should be substantially raised, while T_{m2} and T_{m3} should be slightly reduced. Further, the ratios should satisfy (10).

Figure 3 shows that these conclusions are justified only to a certain extent; the largest deviations occur for point 3 at the level of the heat source, where T_{m3} is slightly higher than for the stagnant case.

If we neglect these distortions and calculate the Peclet criterion for points 1 and 2, we get

$$\text{Pe} = \frac{rv}{a} = \ln \frac{T_{m1}}{T_{m2}} = \ln \frac{2.3}{0.6} = 1.2$$

which is very close to the theoretical value.

As the sand becomes fluidized at u of about 9-10 cm/sec, the deviation from the theoretical scheme of Sec. 1 increases as u rises above 5.8 cm/sec; on the other hand, when the bed becomes fluidized, the solid begins to participate in the longitudinal heat transport, and this has a bulk specific heat 1000 times larger than that of the gas. This results in a marked rise in Pe = rv/a, and we find a very sharp fall in τ_m at point 1, with a correspondingly marked rise in T_{m1} (detailed figures are not given here). Conversely, at point 2 the rise T_{m2} was virtually unobservable.

The heat transport in a ventilated granular bed is characterized by the following two major parameters: the effective thermal diffusivity a and the effective speed v. The gas flow produces some rise in the effective thermal conductivity perpendicular to the flow even at small Pe. The heat transport along the flow on an appreciable scale (Pe = $vL/a \gg 1$) is produced virtually by the flow itself. The influence from the effective thermal diffusivity on the longitudinal transport need be incorporated only at a small distance from the source (Pe ≈ 1). The solid participates in the convective heat transfer when the bed becomes fluidized, and the circulation in the vessel has then to be considered.

NOTATION

x, y, z, τ, coordinates; τ, time; T, temperature; c, specific heat; ρ, density; λ, thermal conductivity; a, thermal diffusivity; u, flow velocity; v, rate of heat removal; Q, heat generated by instantaneous heat source; d, grain diameter; ν, kinematic viscosity; ε, bed porosity; Pe, Peclet number; Re, Reynolds number; Ar, Archimedes number. Indices: g, gas; s, solid; b, bulk; c, critical; e, effective; m, maximum.

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